

## Implementation of Fir Filter Using a Novel modulo Adder for $2^n-2^k-1$ Residue Number System

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**Abstract:** Modular adder is used in Residue Number System (RNS) addition. Moduli set with the form of  $2^n-2^k-1$  ( $1 \leq k \leq n-2$ ) can offer excellent balance among the RNS channels for multi-channel RNS processing. In this paper, a Finite Impulse Response (FIR) filter using a novel algorithm and its Very Large Scale Integration (VLSI) implementation structure are proposed for modulo  $2^n-2^k-1$  adder. In the Modular adder algorithm, parallel prefix operation and carry correction techniques are adopted to eliminate the re-computation of carries. Any existing parallel prefix structure can be used in the novel modulo adder. Thus, we can get flexible tradeoff between area and delay. This paper presents FIR filter implementation with modulo  $2^n-2^k-1$  adder and compared to the conventional FIR filter. This method improves speed, reduces power, delay and area.

**Key words:** RNS, modulo adder, LFSR, FIR filter

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### I. Introduction

RNS is a non-weighted numerical representation system and has carry-free property in multiplication and addition operations. Recent days, it has received intensive study in the VLSI circuits design for digital signal processing (DSP) systems with high speed and low power consumption [2].

For integers A and B, which are of n-bit width, addition is performed [1].

$$C = \langle A + B \rangle_m = \begin{cases} A + B + T < 2^n \\ \langle A + B + T \rangle_n, A + B + T \geq 2^n \end{cases}$$

In the above equation, one of the outputs is selected by given condition. The effective modulo adders in RNS are  $2^n-1$ ,  $2^n$ ,  $2^n+1$ . These  $2^n-1$  and  $2^n+1$  adders are based on parallel prefix and carry correction respectively. Some modulo  $2^n-2^k-1$  adder based on the technique of carry offset, which is only required to obtain the carry information of A+B. In order to find the carries for addition each carry is modified to the utmost carry [4]. By using carry computation, the block corrects the carries further in the proposed modulo adder. In this paper, the modulo adder  $2^n-2^k-1$  is based on the carry correction and parallel prefix addition is proposed. This modulo adder is divided into four units pre-processing unit, the prefix computation unit, the carry correction unit, and the sum computation unit.

This paper is organized as follows: section II describes the RNS background and arithmetic operation. Section III describes the novel modulo  $2^n-2^k-1$  adder and its self-testing. Section IV describes the application of modulo adder in FIR filter. In section V simulation results and comparisons are presented and in section VI conclusion are listed.

### II. Related Work

Shang Ma, Jian-Hao Hu, and Chen-Hao Wang proposed "A Novel Modulo  $2^n-2^k-1$  Adder for Residue Number System". In this system the addition operation is performed using the modulo  $2^n-2^k-1$ . The sum computation operation is done in the adder, which consists of four units, which eliminates re-computation of carries. This system provides excellent performance and reduces the delay and area, thereby reducing the power.

This paper explains the implementation of FIR filter using the novel modulo  $2^n-2^k-1$  adder to show the better performance compared to standard adders used in the filter. Also, the same modulo adder is used in Built In Self Test (BIST) for self-testing using the Linear Feedback Shift Registers (LFSR).

### III. RNS background and Arithmetic Operation.

An RNS is defined by a set of relatively prime integers called the *moduli*. Each integer X can be represented as a set of smaller integers called the residues [3]. This relation can be notationally written based on the congruence:

$$X \bmod m_i = r_i$$

The RNS is capable of uniquely representing all integers X that lie in its *dynamic range*. The dynamic range is determined by the moduli-set  $\{m_1, m_2, \dots, m_n\}$  and denoted as M where,

$$M = \prod_{i=1}^n m_i$$

**Algebraic Operations**

Addition and subtraction of different numbers in the RNS representation is done by individually adding or subtracting the residues with respect to the corresponding moduli.

Consider the moduli-set  $S = \{m_1, m_2, \dots, m_n\}$  and the numbers A and B are given in RNS representation:

$$X = \{A_1, A_2 \dots A_n\} \text{ and } Y = \{B_1, B_2 \dots B_n\}$$

Then,

$$Z = A \mp B = \{z_1, z_2 \dots z_n\}$$

Where  $z_i = (A_i \mp B_i) \bmod m_i$

This property may be applied to subtraction as well, where subtraction of B from A is considered as the addition of  $\bar{B}$ .

The modulo operation is distributive over addition or subtraction:

$$|A \mp B|_m = ||A|_m \mp |B|_m|_m \tag{1}$$

**IV. Modulo  $2^n-2^k-1$  adder**

The novel modulo  $2^n-2^k-1$  structure is shown in figure(1), which consists of four modules, pre-processing unit, carry generation unit, carry correction unit, and sum computation unit. The modulo adder divided into two binary adders with a carry correction and sum computation model, to get correction carries  $C_i^{real}$  find from the process carries in preprocessing unit. At last to get the final modular addition with  $C_i^{real}$  and partial sum information [1].

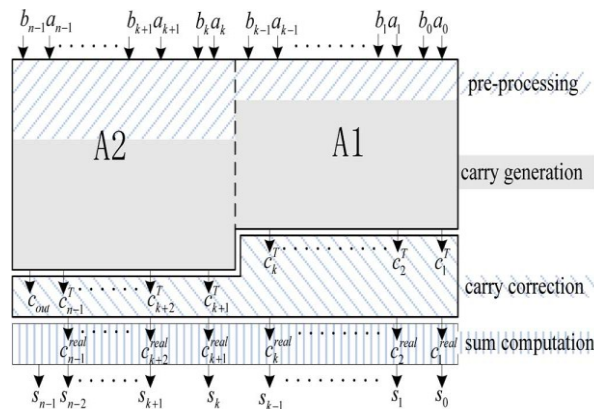


Fig.1: Modulo  $2^n-2^k-1$  adder structure.

**A. Pre-processing Unit**

The preprocessing unit is used for generating carries and carry propagation bits. The computation can be performed by A1 and A2 where A1 and A2 are used for lower k- bits and higher n-k bits addition respectively [1].

For lower k-bits carry propagation is calculated as

$$(g_0, p_0) = (a_0 + b_0, a_0 \oplus b_0) i = 0 \tag{3}$$

$$(g_i, p_i) = (a_i b_i, a_i \oplus b_i) i = 1, 2, \dots, k - 1 \tag{4}$$

For A2(n-k bits) first find by simple carry save adder process

$$(g'_i, p'_i) = (a_i b_i, a_i \oplus b_i) \tag{5}$$

This  $g'$  and  $p'$  are inputs to next stage in A2 part addition. Then the output carry propagation for A2 is

$$(g_k, p_k) = (p'_k, p'_k) i = k \tag{6}$$

$$(g_i, p_i) = (p'_i g'_{i-1}, p'_i \oplus g'_{i-1}) i = k + 1, \dots, n - 1 \tag{7}$$

The carry out for first unit is  $C_{SCSA}$

$$C_{SCSA} = a_{n-1} b_{n-1} = g'_{n-1} \tag{8}$$

**B. Carry Generation**

In the carry generation unit, carry is generated by carry look-ahead adder [1]

$$C'_{i+1} = G_i + P_i C_i \quad (9)$$

**C. Carry Correction Unit**

The carry correction unit is used to get the real carries  $C_{i+1}^{real}$  for each bit needed for final sum generation [1] from equation 10.

$$C_{i+1}^{real} = \begin{cases} C_{i+1}(C_{out} + \overline{P_{i:0}}) & i = 0, 1, \dots, k-1 \\ C_{i+1}(C_{out} + \overline{P_{k-1:0}}(p_k \oplus C_k)) & i = k \\ C_{i+1}(C_{out} + \overline{P_{i:k+1}} + \overline{P_{k-1:0}}(p_k \oplus C_k)) & i = k+1, \dots, n-2 \end{cases} \quad (10)$$

**D. Sum Computation**

The final sum is calculated by the real carries generated from the carry correction unit. The final sum is [1]

$$S_i = C_i^{real} \oplus p_i \quad (11)$$

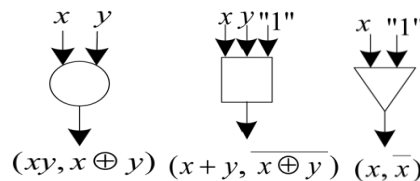
The sum bits are

$$S_i = \begin{cases} C_{out} \oplus \overline{p_0} & i = 0 \\ C_k^{real} \oplus C_{out} \oplus \overline{p_k} & i = k \\ C_i^{real} \oplus p_i & i = 1, \dots, k-1, k+1, \dots, n-1 \end{cases} \quad (12)$$

**E. VLSI Implementation**

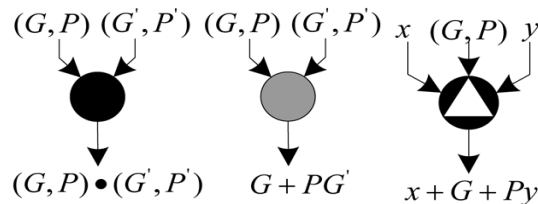
**Pre-Processing Unit:**

The pattern “○” is the pre-processing unit and used to generate carry generation and carry propagation bits for the following prefix computation. Since there are fixed “1” inputs at the 1st and the 4th places, the patterns “▽” “□” are used for this special situations. The pattern “▽” does not cost any resource in unit-gate model.



**Carry Generation Unit:**

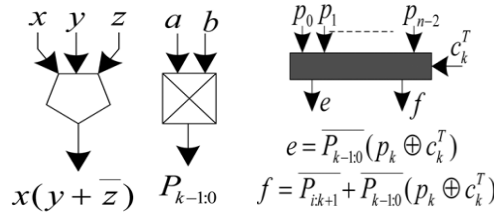
The pattern “●” is the prefix computation unit. In this example, the Sklansky prefix tree is used and there are 11 prefix computation units. The delay of “●” is determined by its carry generation path which is one OR gate and one AND gate. However, the pattern “◐” in the final stage of prefix tree is not needed to compute propagation bits. This is computed by pattern “◑”.





**Carry Correction Unit:**

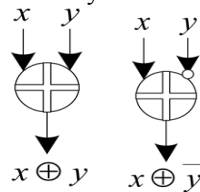
The pattern “◑” performs the computation. Corresponding correction operators are used. There are three different situations, that is  $i = 0, 1, \dots, k-1$ ,  $i = k$  and  $i = k+1, \dots, n-2$ . The  $P_{i:0}$ ,  $z_1$  and  $z_2$  can be computed.  $P_{i:0}$ ,  $z_1$  and  $z_2$  by independent modules. The pattern “◑” and “◒” is used to compute  $C_k^T$  is computed out before  $C_i^T$  ( $i = k+1, \dots, n-1$ ) with two prefix computation stages. Hence, we can get and

without extra delay. In the worst case, the group propagation bits required are needed to be computed one by one from  $p_i (i = 0, 1, \dots, n - 2)$ . However, the extra components for computing these group propagation bits can be removed when the group propagation bits exist in prefix structure.



**The Sum Computation:**

The pattern “” is used for performing the sum computation. As a matter of fact, this operator is the logic XOR operation. The pattern “” is a modified XOR operator, one of its inputs is inverted. Because the computation can be performed with carry correction simultaneously, only one XOR operations are required to perform the sum computation and no extra delay is introduced.



**F. Example Of Modulo Adder**

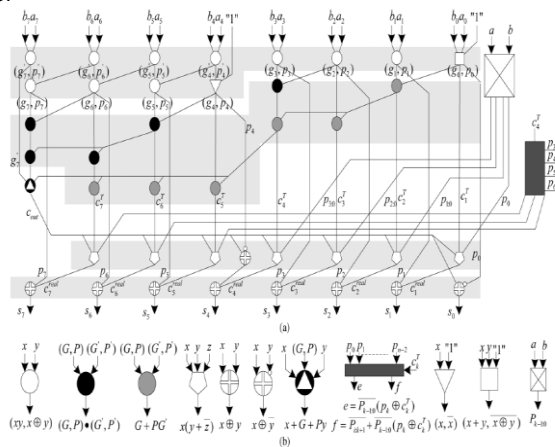


Fig.2:Example of modulo  $2^8-2^4-1$  structure

**Numerical Example**

The computation is performed two stages A1 and A2.

where A1 is for 0 to k bits i.e. lower four bits

where A2 is k to (n-1) bits i.e. higher four bits.

For A1, additions (lower bits addition) carry generation and carry propagation from (3)& (4)

$$\{g_0, g_1, g_2, g_3\} = \{1, 0, 0, 0\}$$

$$\{p_0, p_1, p_2, p_3\} = \{1, 1, 1, 0\}$$

For adder A2, sum of carry propagation of A2 with previous addition A1 output carry. First we add using simple carry save adder for A2 bits from (5)

$$\{g_4, g_5, g_6, g_7\} = \{1, 0, 0, 1\}$$

$$\{P_4, P_5, P_6, P_7\} = \{0, 1, 1, 0\}$$

For simple carry save addition add with previous carries from (6)& (7)

$$\{g_4, g_5, g_6, g_7\} = \{0, 1, 0, 0\}$$

$$\{p_4, p_5, p_6, p_7\} = \{1, 0, 1, 0\}$$

In the carry generation unit, we find the carries using prefix tree adder or carry look- ahead adder

$$C_{i+1} = G_i + P_i C_i$$

Similarly  $\{C_1, C_2, C_3, C_4, C_5, C_6, C_7\} = \{1, 1, 1, 0, 0, 1, 1\}$

And from (9)  $C_{out} = C_7 + G_7 = 1$ .

The carry correction unit is used for get the real carries  $C_{i+1}^{real}$  for each bit needed for final sum generation from (10)

$$\{C_1^{real}, C_2^{real}, C_3^{real}, C_4^{real}, C_5^{real}, C_6^{real}, C_7^{real}\} = \{1, 1, 1, 0, 0, 1, 1\}$$

The sum bits are from (12)

$$\text{Then } \{S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7\} = \{1, 0, 0, 1, 1, 0, 0, 1\} = 153$$

Manually,

$$\left| \begin{array}{r} 215+177 \\ \hline 239 \end{array} \right| = 153$$

**G. Self-Testing**

A built-in self-test (BIST) is a mechanism that allows a machine to test itself. The purpose of BIST is to reduce the complexity, and thereby decrease the cost and reduces the dependence upon external (pattern-programmed) test equipment. A random number generator is a computational or physical device designed to generate a sequence of number that lack any pattern i.e. appear random. Here, in this paper we are generating random numbers using the LFSR.

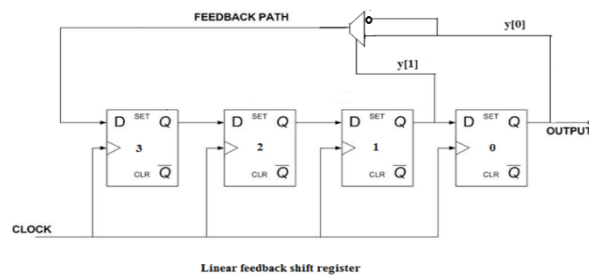


Fig.3: LFSR

The random number generated from the LFSR register is connected to the modulo adder shown below

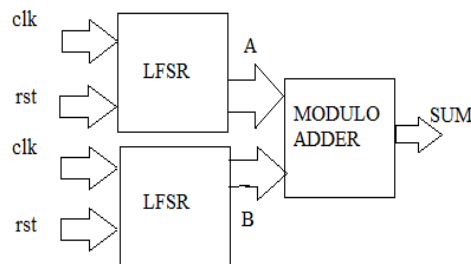


Fig.4: Self-testing modulo adder

We use two LFSR’s for random generation, to produce two inputs A and B to modulo adder. Randomly generated 8-bit outputs are connected to the modulo adder inputs. The sum or the output of MODULO adder is then checked with theoretical values. If those values matches the MODULO adder output it is considered working properly, otherwise not. By using the LFSR we can self-test the MODULO adder

**V. Implementation Of Modulo Adder In Fir Filter**

In signal processing, a finite impulse response (FIR) filter is a filter whose impulse response (or response to any finite length input) is of finite duration, because it settles to zero in finite time. A FIR filter is designed by finding the coefficients and filter that meet certain specifications, which can be in time-domain or frequency-domain.

In this chapter we implement FIR filter with a modulo  $2^n - 2^k - 1$  adder.

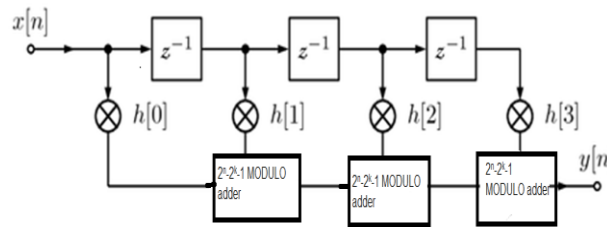


Fig.5: FIR filter with modulo  $2^n - 2^k - 1$  adder

In this, the input signal is multiplied with the coefficients and single clock delay is added through the adder. The adder which is used as the modulo  $2^n - 2^k - 1$  adder. The main advantage of using this adder in the filter is that, it reduces the computation time. It also reduces area, delay, power and complexity. It improves speed of FIR filter.

### VI. Simulation Results

The simulation is done in Xilinx's 14.4 tool. The results of the modulo  $2^n - 2^k - 1$  adder as shown in figure. 6 and 7 and figure 8 shows the result of self-testing of modulo adder using LFSR. The application of FIR filter using modulo adder is shown in figure 9.

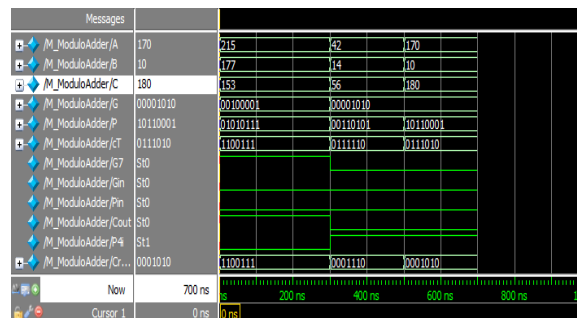


Fig.6: Simulation results of modulo adder

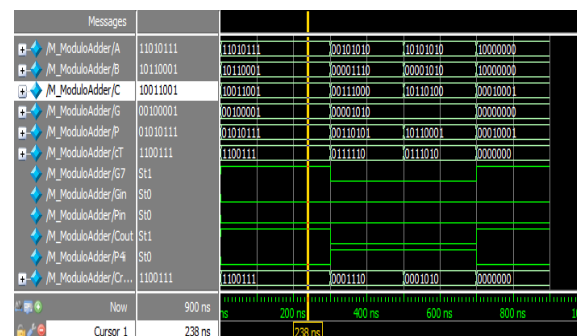


Fig.7: Simulation results of modulo adder

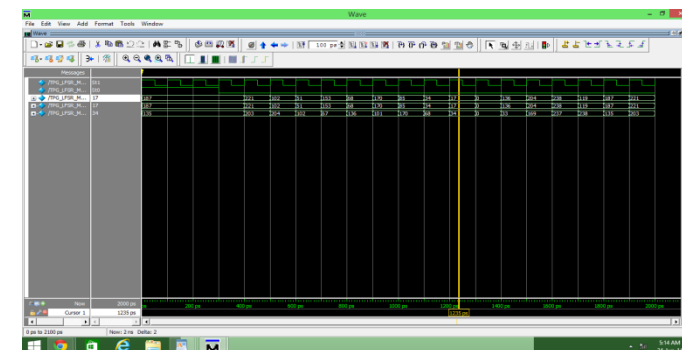


Fig.8: Simulation results of self-testing modulo adder with LFSR

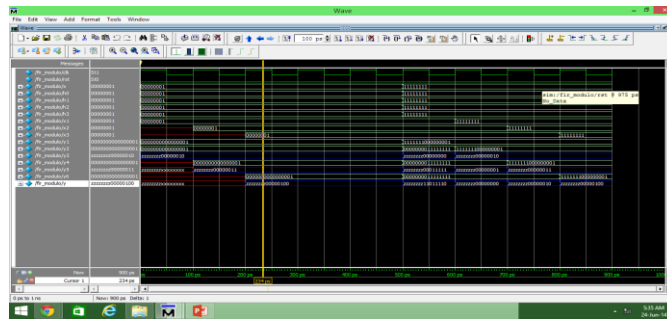


Fig.9: Simulation results of FIR filter using modulo adder

Comparison between conventional FIR filter and FIR filter with modulo adder is shown below in table 1. The modulo FIR filter has a delay of 11.21ns which is much less than conventional FIR filter. The number of registers utilized is just 12 which depicts that the area is less and in turn reduces the power consumption.

**Comparison**

	DELAY	NUMBER OF REGISTERS
CONVENTIONAL FIR FILTER	26.53 ns	44
FIR FILTER WITH MODULO ADDER	11.21 ns	12

Table 1: Comparison of conventional and modulo FIR filter

**VII. Conclusion**

In this paper, a FIR filter is implemented using RNS modulo  $2^n-2^k-1$  adder. The novel modulo adder structure consists of pre-processing unit, carry generation unit, carry generation unit and sum computation unit. This modulo adder has given higher performance in area and delay compared to general modulo adder. Implementation of efficient FIR filter using modulo  $2^n-2^k-1$  is completed with a delay of 11ns i.e. very less delay than the standard FIR. By comparing this with conventional FIR filter, it has given high speed, less power, less delay and area and is much efficient.

**Acknowledgement**

This work is carried out on the basis of the paper entitled “A Novel Modulo  $2^n - 2^k - 1$  Adder for Residue Number System”. We wish to offer our sincere gratitude and thanks to Shang Ma, Jian-Hao Hu, and Chen-Hao Wang for having motivated us to take up the problem of modulo  $2^n - 2^k - 1$  adder.

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